Theme

"Think deeply about simple things"

STEM Innovation Academy Unit 3

Subject: Calculus Unit Title: Applications of Derivatives Grade: 11	Teacher: Ahmed Salama Duration: 8 weeks

Summary of Unit

In this unit student will apply derivatives to find extreme values of functions, to determine and analyze the shapes of graphs, and to solve equations numerically. We also introduce the idea of recovering a function from its derivative. The key to many of these applications is the Mean Value Theorem, which connects the derivative and the average change of a function.

Textbook: Thomas' Calculus Early Transcendentals (Fourteenth Edition)
Online resource: MyMathLab www.mymathlab.com

Stage 1 – Desired Results

Essential Questions:

- What role of derivatives and limits play as a foundation for the calculus and in practical applications?
- What are derivatives used for?
- Why are there so many different rules for differentiation?

Enduring Understandings:

- The derivative has both theoretical and real life applications
- The derivative provides useful information about the behavior of functions and the shape of graphs
- The rate of change of a function allows you to predict future behavior
- The meaning of rates of change in real-life setting.
- Derivative can be used to solve optimization problems

Objectives:

After completing the learning of this unit, student will be able to

- locate and identify extreme (local & absolute maximum or minimum) values of a function from its derivative.
- understand Rolle's Theorem, Mean Value Theorem
- use Mean Value Theorem to reason how two functions are related if they have identical derivatives oven an interval
- find velocity and position from acceleration
- use the First Derivative Test to determine where a function is increases or decreases
- test the critical points of a function to identify if local extreme values are present
- use the second derivative to determine how the graph of a differentiable function bends or turns.

- Sketch graphs and revealing visually the key features of functions by using all ideas from unit 1, 2 and 3
- understand what is indeterminate form and L'Hopital's rule
- use derivatives to solve a variety of optimization problems in mathematics, physics, economics, and business
- understand what is Newton's method (Newton-Raphson method) and apply the technique to approximate the solution to an equation f(x)=0
- understand antiderivatives are the link connecting the two major elements of calculus: derivatives and definite integrals.
- Find antiderivative form a function

Standards/Outcomes: Collage Board

- D2C1: Approximate the derivative. Graphically by finding the slope of a tangent line drawn to a curve at a given point. Numerically by using the difference quotient

- D2C2: Find the equation of the tangent line using the definition of derivative

- D2C5: Apply the Mean Value Theorem on a given interval

- D2C9: Find the equation of:
    - A line tangent to the graph of a function at a point
    - A normal line to the graph of a function at a point

- D2C10: Solve application problems involving:
    - Optimization
    - Related rates

- D2C11: Interpret the derivative as a rate of change and varied applied contexts, including velocity, speed, and acceleration.

Unit Math Practice Standards:

- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model with mathematics.
- **MP.5** Use appropriate tools strategically.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.
- **MP.8** Look for and express regularity in repeated reasoning.

Stage 2 – Assessment Evidence	
Performance Task(s): • Performance Task 1 Use related rate concepts to find change of volume of water in cylindrical flask with change of height. • Performance Task 2: Giving function's curve find linearization at a point. • Performance Task 3: Giving a function , find critical point so we could study local maximum. **Authentic Experiences:** • Find solid revolution by knowing the area of base to a figure. • Given an equation that represent displacement of a flying object find its velocity and acceleration at specific time using first and second derivative. • Showing cylindrical tank is filling with oil, find the change of volume when height change. With respect to time.	**Unit Pre-Assessment:** • Teacher created pre-assessment **Summative Assessment:** • NJIT common Midterm Exam III
Extensions (Tier I): • Enrichment Question (challenging add-ons) • More applications of findings	**Differentiation (Tiers 2 and 3):** • Grouping students (cooperative learning) • Small group support or re-teaching • MyMathLab online work support

Stage 3 – Learning Plan

Vocabulary

- Extreme Values
- Absolute maximum/global Maxima, Absolute minimum/global minima
- Local maximum/minimum
- Fist Derivative Theorem
- Rolle's Theorem
- Mean Value Theorem
- Monotonic functions
- Second Derivative Test
- Point of inflection
- Indeterminate forms
- L'Hopital's Rule
- Cauchy's Mean Value Theorem
- Optimization Problems
- Snell's Law, Law of Refraction
- Newton's Method
- Arbitrary constant

Learning Materials:
Textbook: Thomas' Calculus Early Transcendentals (Fourteenth Edition)

Section 4.1: Extreme Values of Functions (Page 221-229)
Section 4.2: The Mean Value Theorem (Page 229-237)
Section 4.3: Monotonic Functions and the First Derivative Test (Page 237 – 242)
Section 4.4: Concavity and Curve Sketching (Page 242 -255)
Section 4.5: Indeterminate Forms and L'Hopital's Rule (Page 255 – 264)
Section 4.6: Applied Optimization (Page 264 – 276)
Section 4.7: Newton's Method (Page 276 -280)
Section 4.8: Antiderivatives (Page 281 – 291)